

中國文化大學 九十五 學年度 第二學期 期中 考試試卷					
考試科目	任課老師	系級	考試日期	份數	備註
工程數學	陳為仁	機二 A	95/04/20	60	Open A4 note

- Given three points  $A = (1, 1, 1)$ ,  $B = (2, 2, 2)$ ,  $C = (6, 1, 3)$  and  $D = (-2, 4, 6)$ .
  - Find the cosine of the angle between  $\overline{AB}$  and the line from A to the midpoint of  $\overline{BC}$ .
  - Prove that the three points A, B and C are not collinear.
  - Find an equation of the plane containing all three points A, B and C.
  - Find the volume of the parallelepiped having incident sides extending from the first point A to each of the other three points B, C and D.
- In each of problems given, determine whether the set of vectors is a subspace of  $\mathbb{R}^n$  for the appropriate n.
  - A set S consists of all vectors in  $\mathbb{R}^4$  of the form  $(x, y, x+y, x-y)$ .
  - A set S consists of all vectors in  $\mathbb{R}^6$  of the form  $(x, 0, 0, 1, 0, y)$ .
- Determine whether the given vectors  $(1, -2)$ ,  $(4, 1)$  and  $(6, 6)$  are linearly independent or dependent in  $\mathbb{R}^2$ .

3. Given a matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

- Find the reduced form  $A_R$  of matrix A .
- Determine the rank of matrix A and state the reason.
- Determine a basis for the row space of matrix A and its dimension.
- Determine a basis for the column space of matrix A and its dimension.

5. Given a linear system  $\mathbf{AX} = \mathbf{B}$  where  $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & 1 \end{bmatrix}$ ,  $\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$  and  $\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ .

- Determine the dimension of the solution space of the homogeneous system  $\mathbf{AX} = \mathbf{0}$  if matrix  $\mathbf{B} = \mathbf{0}$ .
- Show that the nonhomogeneous system  $\mathbf{AX} = \mathbf{B}$  has a solution if  $\mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .
- Find the general solution of nonhomogeneous system in (2).