1. Given three points A = (1, 0, -2), B = (0, 0, 0) and C = (5, 1, 1).

    (1) Find the cosine of the angle between $\overline{AB}$ and the line from A to the midpoint of $\overline{BC}$.

    (2) Verify if $\overline{AB}$ and $\overline{BC}$ are orthogonal.

    (3) Prove that the three points are not collinear.

    (4) Find an equation of the plane containing all three points.

    (5) Find the area of the parallelogram having incident sides extending from the first point to each of the other two..

2. Given a set S consisting of all vectors (x, y, x+y, x-y) in R^4.

    (1) Prove that the set S is a subspace of R^4.

    (2) Determine a basis and the dimension for the subspace S.

    (3) Prove that the vectors forming a basis for S are linearly independent.

    (4) Determine the dimension for the subspace S.

3. Given a matrix $A = \begin{bmatrix} 1 & -1 & 4 & 2 \\ 0 & 1 & 3 & 2 \\ 3 & -2 & 15 & 8 \end{bmatrix}$

    (1) Find the reduced form A_R of matrix A .

    (2) Determine the rank of matrix A and state the reason.

    (3) Determine a basis for the row space of matrix A and its dimension.

    (4) Determine a basis for the column space of matrix A and its dimension.

5. Given a linear system $\mathbf{AX} = \begin{bmatrix} -1 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

    (1) Determine the dimension of the solution space of the homogeneous system **AX=O** if matrix **B=O**.

    (2) Show that the nonhomogeneous system AX=B has a solution if $\mathbf{B} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

    (3) Find the general solution of nonhomogeneous system in (2).

5. Given two square matrices $A = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

    (1) Determine whether the matrix is singular or nonsingular. Why?
    (2) Find the inverse of the matrix if it is nonsingular.
    (3) Determine whether the matrix AB has an inverse or not.