1. Given a square matrix $A = \begin{bmatrix} -1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$.

   (1) Find its eigenvalues and the associated eigenvectors. (14)

   (2) Find a matrix **P** that diagonalizes matrix **A**. (3)

   (3) $\mathbf{P^{-1}AP} = ?$ (3)

2. Given a quadratic form $F = x_1^2 - 2x_1 x_2 + x_2^2$.

   (1) Find a matrix A such that quadratic form is X^tAX with $X^t=[x_1 \ x_2]$. (5)

   (2) There exits a coordinate transformation X=QY which transforms the given F to the standard form. Find the matrix Q and the standard form. (6)

   (3) Show that matrix Q in (2) is an orthogonal matrix. (3)

3. Find the general solution of the following nonhomogeneous system of linear differential equations. (20)
$$x_1' = 3x_1 + 3x_2 + 8$$
$$x_2' = x_1 + 5x_2 + 4$$

4. Given: A curve C has parametric equations

   $x = \cos(t)+t\sin(t)$, $y =\sin(t)-t\cos(t)$, $z = t^2$ for $1\le t \le 5$.

   (1) Find the length function s(t) and calculate the length of curve C. (7)

   (2) Find the curvature κ of curve C. (7)

5. Given a scalar function $\varphi(x, y, z) = xyz$. (12)

   (1) Compute $\nabla \varphi$

   (2) Compute $curl(\nabla \varphi)$.

   (3) Compute $div(\nabla \varphi)$.

6. Find the streamlines of the vector field $\vec{F}(x, y, z) = 1/x\vec{i} + e^x \vec{j} - \vec{k}$. (10)

7. 請回答下列有關矩陣的性質，正確者答「是」，反之答「否」。(注意：答錯者該子題除不予計分外，並倒扣 2 分) (10)

   (1) Matrix A is an orthogonal matrix if and only if $A^{-1} = A^t$.

   (2) Matrix A is a symmetric matrix if and only if $A = A^{-1}$.

   (3) The eigenvalues of a real symmetric matrix are real.

   (4) A complex matrix U is unitary if and only if $U^{-1} = U^t$.

   (5) A complex matrix H is hermitian if and only if $\overline{H} = H^t$.