

中國文化大學 九十三 學年度 第二學期 期末 考試試卷					
考試科目	任課老師	系級	考試日期	份數	備註
工程數學	陳為仁	機二 B	94/06/22	80	close books

1. Given a square matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(1) Find its eigenvalues and the associated eigenvectors.

(2) Show if the matrix A is diagonalizable.

2. Given a quadratic form $F = x_1^2 + 2x_1x_2 + 6x_2^2$.

(1) Find a matrix A such that quadratic form is X^tAX with $X^t=[x_1 \ x_2]$.

(2) Show that matrix A is a symmetric matrix.

(3) Find the standard form of the quadratic form.

3. Find the general solution of the following homogeneous system of linear differential equations.

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

4. Given: A curve C has parametric equations $x=2t^2$, $y=3t^2$, $z=4t^2$ for $1 \leq t \leq 3$.

(1) Write the position vector $\vec{F}(t)$ and tangent vector $\vec{F}'(t)$ for curve C.

(2) Find the length function $s(t)$ and calculate the length of curve C.

(3) Find the curvature κ of curve C.

5. Given a vector field $\vec{F}(x, y, z) = e^z\vec{i} - x^2\vec{k}$.

(1) Compute $\text{curl}\vec{F}$.

(2) Compute $\text{div}\vec{F}$.

(3) Find the streamlines of the vector field \vec{F} .

6. Calculate the work done by the force $\vec{F} = \vec{i} - y\vec{j} + xyz\vec{k}$ in moving a particle from $(0, 0, 0)$ to $(1, -1, 1)$ along the curve $x = t$, $y = -t^2$, $z = t$ for $0 \leq t \leq 1$.