1. Given: Function f(x) is defined on the given interval

    $$f(X) = X \qquad -\pi \le X \le \pi$$

    Find: (1) Find the Fourier series of f(x).

    (2) Use the result in (1) to show that $\ 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots = \dfrac{\pi}{4}$

2. Given: $f(x) = x, \qquad 0 \le x \le 1$

    (1) Expand f(x) in Fourier cosine series with period 2.

    (2) Expand f(x) in Fourier sine series with period 2.

3. Given the Sturm-Liouville problem. $\ y'' + \lambda y = 0 \qquad y(0) = 0, \qquad y'(\pi) = 0$

    (1) Find the eigenvalues and corresponding eigenfunctions.

    (2) Prove the set of eigenfunctions obtained in (1) is orthogonal on $\ 0 \le x \le \pi$.

    (3) Find the norm for each eigenfunction.

4. Find the temperature u(x,t) for a bar with boundary conditions and initial condition as given.

    $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad 0 < x < \pi, \quad 0 < t < \infty$$

    $$\frac{\partial u}{\partial x}(0,t) = 0, \quad \frac{\partial u}{\partial x}(\pi,t) = 0, \quad 0 < t < \infty$$

    $$u(x,0) = \sin x \qquad 0 < x < \pi$$

5. Find the steady-state temperature u(x,y) for a rectangular plate with boundary conditions as given.

    $$\frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad 0 < X < a, \quad 0 < y < b$$

    $$\left. \frac{\partial u}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial u}{\partial x} \right|_{x=a} = 0, \quad 0 < y < b$$

    $$u(x,0) = 0, \quad u(x,b) = 1, \quad 0 < x < a$$