

中國文化大學 九十四學年度 第二學期 期中 考試試卷					
考試科目	任課老師	系級	考試日期	份數	備註
高等工數	陳為仁	機三 B	95/04/11	70	close books

1. Given: Function $f(x)$ is defined on the given interval

$$f(x) = x \quad -\pi \leq x \leq \pi$$

Find: (1) Find the Fourier series of $f(x)$.

(2) Use the result to show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$

2. Given: $f(x) = 4, \quad 0 \leq x \leq 3$

(1) Write the Fourier cosine series of the given function on the interval.

(2) Write the Fourier sine series of the given function on the interval.

3. Given the Sturm-Liouville problem. $y'' + \lambda y = 0 \quad y'(0) = 0, \quad y(4) = 0.$

(1) Find the eigenvalues and eigenfunction of the given boundary-value problem.

(2) Prove the set of eigenfunctions obtained in (1) is orthogonal on $0 < x < 4$.

(3) Find the norm for each eigenfunction.

4. Find the temperature $u(x,t)$ for a bar with boundary conditions as given.

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \quad 0 < x < L, \quad 0 < t < \infty$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad \frac{\partial u}{\partial x}(L,t) = 0, \quad 0 < t < \infty$$

$$u(x,0) = f(x) \quad 0 < x < L$$

5. Find the twist angle $\theta(x,t)$ for a bar with boundary conditions as given

$$a^2 \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial t^2} \quad 0 < x < 1, \quad t > 0$$

$$\theta(0,t) = 0, \quad \theta(1,t) = 0, \quad t > 0$$

$$\theta(x,0) = x, \quad \left. \frac{\partial \theta}{\partial t} \right|_{t=0} = 0 \quad 0 < x < 1$$