1. Given: Function f(x) is defined on the given interval

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \pi - x & 0 < x < \pi \end{cases}$$

   Find: (1) Find the Fourier series of f(x).   (20)

   (2) Use the result to show that $1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \cdots = \dfrac{\pi^2}{8}$ (5)

2. Given the set of functions {1, cos(x), cos(2x),…}.

   (1) Show that the given set of functions is orthogonal on interval [-π,π].   (15)

   (2) Find the norm of each function in the set.   (10)

3. Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem.

   $$y'' + \lambda y = 0 \qquad y(\pi) = y(-\pi), \qquad y'(\pi) = y'(-\pi) \qquad (20)$$

4. Find the twist angle θ(x,t) for a bar with initial and boundary conditions as given

   $$a^2 \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial t^2} \qquad 0 < x < 1, \quad t > 0$$

   $$\theta(0,t) = 0, \qquad \frac{\partial \theta}{\partial x}(1,t) = 0, \quad t > 0$$

   $$\theta(x,0) = x \qquad , \qquad \left. \frac{\partial \theta}{\partial t} \right|_{t=0} = 0 \qquad 0 < x < 1 \qquad (25)$$

5. Find the steady-state temperature u(x,y) for a rectangular plate with boundary conditions as given.

   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad 0 < x < a, \quad 0 < y < b$$

   $$\left. \frac{\partial u}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial u}{\partial x} \right|_{x=a} = 0, \quad 0 < y < b$$

   $$u(x,0) = 0, \quad u(x,b) = f(x), \quad 0 < x < a \qquad (25)$$